Cambridge International AS Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary Level

MATHEMATICS **9709/21**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Use logarithm subtraction property to produce logarithm of quotient	**M1**	
	Factorise at least as far as $x(x^2-4)$ and $x(x-2)$ or use correct algebraic long division to obtain a quotient of $x+2$ and a remainder of 0 from correct working	**B1**	Allow B1 either before or after application of log property Allow B1 for equivalent using factorisation then use of addition rule Allow B1 for $\frac{(x+2)(x^2-2x)}{(x^2-2x)}$
	Obtain final answer $\ln(x+2)$ using correct process	**A1**	With no errors seen
		3	

Question	Answer		Guidance
2(i)	State or imply non-modular inequality $(3x-5)^2<(x+3)^2$ or corresponding equation or pair of different linear equations/inequalities	**B1**	**SC**: Allow B1 for $x<4$ from only one linear inequality
	Attempt solution of 3-term quadratic equation/inequality or of two different linear equations/inequalities	**M1**	For M1, must get as far as 2 critical values
	Obtain critical values $\frac{1}{2}$ and 4	**A1**	
	State answer $\frac{1}{2}<x<4$ or equivalent	**A1**	If given as 2 separate statements, condone omission of 'and' or $\cap$ but penalise inclusion of 'or' or $\cup$
		4	

Question	Answer	Marks	Guidance
2(ii)	Attempt to find n (not necessarily an integer so far) from $3^{0.1n} =$ or $<$ *their* positive upper value from part **(i)** or $3^{0.1n+1} =$ or $< 3 \times$ *their* positive upper value from part **(i)**	**M1**	0/2 for trial and improvement
	Conclude 12	**A1**	
		2	

Question	Answer	Marks	Guidance
3	Use product rule to differentiate $x^2 \ln y$	**M1**	Allow M1 for $2x \ln y + x^2 y^{-1}$ oe
	Obtain $2x \ln y + x^2 \times \frac{1}{y} \times \frac{\mathrm{d}y}{\mathrm{d}x}$	**A1**	
	Obtain $\ldots +2 + 5\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**B1**	B1 for $+2 + 5\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, maybe implied by later work
	Substitute $x = 3$ and $y = 1$ to find value of their $\frac{\mathrm{d}y}{\mathrm{d}x}$	***M1**	Dependent on at least one $\frac{\mathrm{d}y}{\mathrm{d}x}$ present
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{14}$	**A1**	
	Attempt equation of line through $(3,\ 1)$ with gradient of normal	**DM1**	Allow one sign error
	Obtain $y = 7x - 20$ or equivalent unsimplified	**A1**	**FT** on their perpendicular gradient
		7	

Question	Answer	Marks	Guidance
4(a)	Use identity $\tan^2 3x = \sec^2 3x - 1$	**B1**	
	Integrate to obtain form $k_1 \tan 3x + k_2 x$	**M1**	
	Obtain correct $\frac{1}{3}\tan 3x - x + c$	**A1**	
		3	
4(b)	Express integrand as $e^{2x} + 4e^{-x}$	**B1**	
	Integrate to obtain form $k_3 e^{2x} + k_4 e^{-x}$	**M1**	
	Obtain correct $\frac{1}{2}e^{2x} - 4e^{-x}$	**A1**	
	Use limits to obtain $\frac{1}{2}e^2 - 4e^{-1} + \frac{7}{2}$ or similarly simplified equivalent	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	Substitute $x = 2$ and equate to zero	**M1**	Allow synthetic division for each– must result in an equation from each division
	Substitute $x = -1$ and equate to 27	**M1**	Allow unsimplified
	Obtain $4a + 2b = -24$ and $a - b = 48$ or equivalents	**A1**	Allow one error in each equation
	Solve a relevant pair of simultaneous linear equations	**M1**	Dependent at least one M mark
	Obtain $a = 12, \quad b = -36$	**A1**	
		5	
5(ii)	Divide by $x - 2$ at least as far as the x term to obtain $5x^2 + (\textit{their } a + 10)x...$	**M1**	For synthetic division need to see 5 and $\textit{their } a + 10$ in the bottom line
	Obtain $5x^2 + 22x + 8$	**A1**	
	Obtain $(x - 2)(5x + 2)(x + 4)$	**A1**	If solved using a calculator and then forming factors, must be correct for full marks
		3	

Question	Answer	Marks	Guidance
6(i)	Use quotient rule (or product rule) to differentiate	**M1**	Penalise missing brackets by withholding the A mark unless recovered later
	Obtain $\frac{dy}{dx} = \frac{3x^2(2-5x)-(-5)(8+x^3)}{(2-5x)^2}$ or equivalent	**A1**	
	State or imply curve crosses x-axis when $x = -2$	**B1**	
	Substitute -2 to obtain 1	**A1**	
		4	
6(ii)	Equate numerator of first derivative to zero and rearrange as far as $kx^3 = ...$ or equivalent	**M1**	
	Confirm given result $x = \sqrt{0.6x + 4x^{-1}}$ **AG**	**A1**	Condone in this part error(s) in denominator of derivative
		2	
6(iii)	Use iterative process correctly at least once	**M1**	
	Obtain final answer 1.81	**A1**	
	Show sufficient iterations to 5 sf to justify answer or show a sign change in the interval [1.805, 1.815]	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	State or imply $\text{cosec}\,2\theta = \dfrac{1}{2\sin\theta\cos\theta}$	**B1**	
	Attempt to express left-hand side in terms of $\sin\theta$ and $\cos\theta$ only	**M1**	
	Simplify to confirm $\text{cosec}^2\theta$ **AG**	**A1**	
		3	
7(ii)	Use identity to express left-hand side in terms of $\sin 30$ or $\text{cosec}\,30$	**M1**	
	Obtain $\dfrac{2}{\sin 30}$ or $2\,\text{cosec}\,30$ and confirm 4 **AG**	**A1**	
		2	
7(iii)	Solve quadratic equation of the form $k\text{cosec}^2\dfrac{\phi}{2} + \text{cosec}\dfrac{\phi}{2} - 12 = 0$ or	***M1**	Allow sign errors
	$12\sin^2\dfrac{\phi}{2} - \sin\dfrac{\phi}{2} - k = 0$ correctly for $\text{cosec}\,\frac{1}{2}\phi$ or $\sin\frac{1}{2}\phi$ to find two values of $\sin\frac{1}{2}\phi$ or $\text{cosec}\,\frac{1}{2}\phi$ Obtain $\sin\frac{1}{2}\phi = -\frac{1}{4}, \frac{1}{3}$	**A1**	
	Use correct process to find at least one correct value of ϕ from $\sin\frac{1}{2}\phi = \pm\frac{1}{4}, \pm\frac{1}{3}$	**DM1**	Allow for any rounded or truncated value
	Obtain any two of –331.0, –29.0, 38.9, 321.1	**A1**	Allow greater accuracy
	Obtain all four values and no others between –360 and 360	**A1**	Allow greater accuracy
		5	